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The initial objective was the proof of the concept, which was accomplished with the following steps.

1. Assume a potential model as the "truth" potential, e.g., a simple potential with 1000 mass points.
2. Generate "truth" satellite orbits from the potential of Step 1.
3. Using the orbits from Step 2, extract "observations". These observations of the "truth" model may be used to calculate a best alternate potential model in a least squares sense. The alternate model is a point mass model with 100, 200, 500, etc. total mass points.
4. Compare the orbits generated from the model potential with the orbits generated from the "truth" potential.

As was reported in September 1996, the proof of the concept phase did validate or refine most of the basic ideas and suggested a clearer future course of action.

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GLOBAL RESONANCE EFFECTS ON DEEP SPACE ORBITS

Professor V. Szebehely
Dr. K. Zare
W. Boyce

The University of Texas at Austin

March 10, 1997.

Analysis of Numerical Results

We begin this part by describing a major difference between circular resonance orbits (geosynchronous, GPS), and an elliptic resonance orbit (MOLNIYA).

As described in our previous reports, the disturbing function in terms of orbital elements is given by

$$R = \sum_{l=2}^{\infty} \sum_{m=0}^{l} \sum_{p=0}^{l} \sum_{q=-\infty}^{\infty} R_{lmpq}$$

where

$$R_{lmpq} = \frac{\mu}{a} J_{lm} \left(\frac{R_e}{a} \right)^l F_{lmp}(i) G_{lpq}(e) \begin{Bmatrix} \cos \\ \sin \end{Bmatrix}_{(l-m)odd}^{(l-m)even}$$
$$\left[(l-2p)\omega + (l-2p+q)M + m(\Omega - \theta - \lambda_{lm}) \right].$$

Here $F_{lmp}(i)$ and $G_{lpq}(e)$ are the inclination and eccentricity functions and are given explicitly by Kaula (1966). Furthermore, $\theta = \omega_e t$ where ω_e is the rotation rate of the earth. Since to the first approximation $M = nt$, $\omega =$ constant and $\Omega =$ constant, the frequencies are given by

$$f_{lmpq} = (l-2p+q)n - m\omega_e$$

and therefore resonance $(f = 0)$ is defined by $\frac{n}{\omega_e} = \frac{m}{l-2p+q}$. This leads to the conclusion that for one-to-one resonance $\left(\frac{n}{\omega_e} = 1 \right)$ any combination of l and m represents resonance. For two-to-one resonance $\left(\frac{n}{\omega_e} = 2 \right)$ only even order harmonics $(m = 2, 4, 6, \ldots)$ of arbitrary degree represent resonance.

$\frac{n}{\omega_e}=1$

order	degree
m=1	l=1,2,...
m=2	l=2,3,...
m=3	l=3,4,...

$\frac{n}{\omega_e}=2$

order	degree
m=2	l=2,3,...
m=4	l=4,5,...
m=6	l=6,7,...

The above harmonics represent resonance for any eccentric orbit such as MOLNIYA orbit with $\frac{n}{\omega_e}=2$. However, for circular orbits $(q=0)$ only a subset of the above harmonics represents resonance. These subsets are defined by $\frac{n}{\omega_e}=\frac{m}{l-2p}$. In particular the subsets for geosynchronous orbits $\left(\frac{n}{\omega_e}=1\right)$, and GPS orbits $\left(\frac{n}{\omega_e}=2\right)$ are given by

$\frac{n}{\omega_e}=1$

order	degree
m=1	l=1,3,5,7,...
m=2	l=2,4,6,8,...
m=3	l=3,5,7,9,...

$\frac{n}{\omega_e}=2$

order	degree
m=2	l=3,5,7,9,...
m=4	l=4,6,8,...
m=6	l=7,9,...

The numerical results in this report (see Appendix) fit well the above description. The average effect of each individual harmonic on the position can be estimated from the difference of two consecutive RMS values. The results for geosynchronous and GPSorbits are presented in Table I and Table II.

The average effects of individual harmonics on the position for a geosynchronous orbit

m ↓ / l →	2	3	4	5	6
1	0.000	0.792	0.000	0.002	0.000
2	7.973	0.000	0.092	0.000	0.000
3	——	0.278	0.002	0.013	0.000
4	——	——	0.181	0.000	0.001
5	——	——	——	0.055	0.000
6	——	——	——	——	0.001

Table II

The average effects of individual harmonics on the position for a GPS orbit.

m ↓ / l →	2	3	4	5	6	7
1	0.000	0.026	0.000	0.000	0.000	0.000
2	0.822	0.043	0.007	0.012	0.000	0.000
3	——	0.045	0.015	0.001	0.000	0.000
4	——	——	0.185	0.005	0.003	0.000
5	——	——	——	0.015	0.003	0.000
6	——	——	——	——	0.000	0.007
7	——	——	——	——	——	0.000

January 28, 1997

Representation of Geopotential with Point Masses

Prepared for the USAF Office of Scientific Research
by
V. Szebehely, K. Zare, and S. Chesley

Concept

The standard formulation of the geopotential is the spherical harmonic expansion using Legendre polynomials. A proposed alternate formulation consists of representing the earth by a large number of point masses positioned on a fixed grid within the geoid. We have three primary reasons to pursue this alternate method:

1. The calculation of gravitational acceleration with the alternate formulation is a simple computation using Newton's law of gravity for each mass. The same calculation with the standard formulation requires the recursive computation of the Legendre polynomials. This strongly suggests that the alternate method may significantly reduce the computation time needed to provide a given accuracy. (Or, equivalently, provide greater accuracy with the same computational expense.) Additionally, the point mass method is ideally suited for parallel computation, while the standard formulation cannot be computed in parallel.
2. The standard formulation has a slow rate of convergence due to resonant terms in the harmonic expansion. This leads to a serious problem for highly eccentric resonant orbits, such as Molnya orbits. We expect higher rates of convergence as well as a generally monotonic convergence for a mass point model. Hence this alternate method should prove more suitable for the challenging case of resonant orbits.
3. The alternate method is also appealing because it is much more physically intuitive. An accurate point mass model would by its very nature provide the three-dimensional density distribution of the earth. Such geophysical information may be used to improve the modern earth models and it would be of interest to specialists outside the field of orbital mechanics.

1996 Accomplishments (Previous Work)

The initial objective was a proof of the concept, which was accomplished with the following steps.

1. Assume a potential model as the "truth" potential, e.g., a simple potential with 1000 mass points.
2. Generate "truth" satellite orbits from the potential of Step 1.
3. Using the orbits from Step 2, extract "observations". These observations of the "truth" model may be used to calculate a best alternate potential model in a least squares sense. The alternate model is a point mass model with 100, 200, 500, etc. total mass points.
4. Compare the orbits generated from the model potential with the orbits generated from the "truth" potential.

As was reported in September 1996, the proof of concept phase did validate or refine most of the basic ideas and suggested a clearer future course of action.

It should be emphasized that the bulk of activity in the first year was spent discussing possible strategies and implementing these strategies on a computer. The writing and testing of software have required a substantial up-front investment of time, and we now have computer implementations for the following tasks.

- Create a point mass model with a prescribed number of points. The model consists of a series of concentric shells of points, each point being associated with an approximately equal volume. The mass of each point is calculated based on a non-uniform density distribution for the earth.
- Generate simulated "truth" observations from a point mass or spherical harmonic potential. These can be obtained for a number of different satellites in various orbit classes. The observations may come in varying time intervals and can have random noise added to the position.
- Estimate the potential model based on selected observations. In practice one must make an initial guess and use very short orbit arcs to refine that guess. Due to the poor nature of the initial guess a long orbit arc goes beyond the linear region and the method fails. As the guesses become more and more refined the orbit arcs used as observations may become longer and longer, thus improving the accuracy of the estimate.

1997 Objectives (Future Work)

The ultimate goal of this research is to obtain a working mass point geopotential model which can be used to accurately compute orbits of earth-orbiting satellites. There are two preliminary issues that need to be considered before this task can be approached:

- The computational cost of estimating the appropriate mass values depends strongly on the number of point masses used in the model. Given a density distribution over the volume of the earth, how many mass points are needed to provide a given level of position accuracy in an orbital simulation? On a more practical level, how many mass points are needed to match the level of accuracy provided by a given truncation of the standard formulation, e.g., a 12×12 expansion? This will depend on the complexity of the density distribution, and to a lesser extent on the class of orbit being analyzed.
- The efficiency of the estimation process is, of course, influenced by the observational data provided to the routine. The choice of the number of different satellites observed, the number of observations for each satellite, and the time interval between observations (length of orbit arc) all affect the expense of the estimation. In fact a poor choice can lead to no convergence. Is there a reliable way of determining the optimal choice of these parameters before the estimation has begun? If not, is there a way of avoiding the decidedly non-optimal choices?

Once these questions have been addressed we will be ready to use precise real-world orbit ephemerides to calculate a working geopotential model. Such a calculation will likely require many mass points which will dictate the use of a supercomputer. One should note that the time consuming estimation process does not need to be done as a part of routine orbit determination. Once a "best" point mass model has been determined, it would be used instead of the standard formulation to streamline any future orbit prediction work.

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